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COMPOSITION, COLLIMATION, CONTAMINATION: THE JET OF CYGNUS X-1

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ABSTRACT

We model the observed size and brightness of the VLBA radio core of the jet in Cygnus X-1 to derive an expression for the jet power as a function of basic jet parameters. We apply this expression to recent constraints on the jet power from observations of a large scale shocked shell around the source by Gallo et al. 2005, which leads us to a set of alternative conclusions: either (a) the jet contains large amounts of protons: ($\gtrsim 2000$ protons per radio emitting electron), (b) it has a very low volume filling factor of $f \lesssim 3 \times 10^{-5}$, (c) the steady, radio emitting VLBA jet is not the source of the kinetic energy powering the ISM shell, or (d) its asymptotic behavior differs fundamentally from a broad set of plausible analytic jet models.

Subject headings: ISM: jets and outflows — X-rays: binaries — radio continuum: general — X-rays: individual (Cygnus X-1)

1. INTRODUCTION

The field of X-ray binary (XRB) study has seen a remarkable transformation over the past 5 years: Relativistic jets have moved from being considered exotic and rare abnormalities to being recognized as integral and maybe vital components in the transfer of energy and angular momentum in accreting stellar mass compact objects. While it has been known for decades that the jets produced by supermassive black holes carry enormous amounts of energy and transform their environment, it has been difficult to make firm estimates of the kinetic power of XRB jets.

Furthermore, the study of XRB jets is plagued by the same dilemma facing extragalactic jet research for decades: a lack of solid information about jet composition (i.e., do jets mostly contain protons or positrons as positive charger carriers, and is the inertia dominated by non-thermal relativistic particles or cold, thermal plasma), jet speed (how relativistic are the jets in their very cores), and the jet power. These questions have been discussed for decades (e.g. Reynolds et al. 1996; Wardle et al. 1998; Ghisellini & Celotti 2001), yet conclusive observational evidence to answer them is sparse at best, so the debate continues. In this situation, tight constraints from individual objects are our best bet at making progress on any of these questions.

The interest in XRB jets has arisen from the discovery of the nearly universal presence of compact, flat spectrum radio emission from accreting black hole X-ray binaries (Fender 2001) in the so-called low/hard state, where most of the X-ray emission is in the form of a hard powerlaw (see McClintock & Remillard 2003, for a detailed review of black hole X-ray states). In two cases, this radio emission has been resolved into clearly collimated jets (Dhawan et al. 2000; Stirling et al. 2001). The spectral and morphological similarity to the cores of AGN jets further strengthens the interpretation of this radio emission as evidence for jets.

However, these jets are not resolved transverse to their axes and in the absence of other information, it has been difficult to constrain the kinetic power of these jet - opening angles have been assumed, but the results are then always subject to an unknown, arbitrary parameter. Furthermore, the information encoded in the spatial distribution of the observed radio emission of the VLBI observations of the two jets which have been resolved (Cygnus X-1 and GRS 1915+105) have not been harvested to a degree that allows tight constraints to be derived. The aim of this paper is to derive just such constraints. Cygnus X-1 is far better suited for such a study because its distance is know to significantly higher accuracy than that of GRS 1915+105.

The status of firm observational data on XRB jet power was also recently improved by the discovery of a shell of thermal emission around Cygnus X-1 (Gallo et al. 2005). This shell has been interpreted as the shocked ISM around a low surface brightness, jet-driven radio lobe. Using analysis borrowed from radio galaxy dynamics, the authors limit the average kinetic power $\langle W \rangle$ from the source to fall between $3\times10^{36}\,\mathrm{ergs\,s^{-1}} < \langle W \rangle < 3\times10^{37}\,\mathrm{ergs\,s^{-1}}$. This discovery, together with the VLBA observations of resolved jet emission, makes Cygnus X-1 an ideal candidate to tighten the constraints on fundamental jet parameters.

In order to put these limits into the context of the compact VLBA jet observed by Stirling et al. (2001), we apply a representative emission model to the VLBA data (§2). This allows us to derive a parameterized expression for the kinetic power which depends only on a few unknowns which we hope to constrain, namely the particle content of the jet, the filling factor of emitting material, and the equipartition fraction of the magnetic field. In §3 we compare the outcome to the results of (Gallo et al. 2005), which is orders of magnitude larger than the estimate we derive for our fiducial set of parameters, and discuss the quantitative constraints that can be derived on the set of interesting parameters from this comparison. Section 4 summarizes our findings.

2. THE EMISSION FROM THE JET IN CYGNUS X-1

The radio spectrum of the jet in Cygnus X-1 is flat to slightly inverted ($\alpha \equiv d\log L_\nu/d\log\nu \gtrsim 0$), with a flux level varying between a few and a few tens of mJy (Brocksopp et al. 1999). Stirling et al. (2001, epochs A and C) resolved about 50% of the emission to be extended, measuring roughly $\mu \sim 10-15$ mas. The canonical model for the flat radio emission from jet cores goes back to Blandford & Koenigl (1979, BK hereafter). This model has been used successfully and extensively in the context of both AGN jet cores and X-ray binary jets (e.g. Hjellming & Johnston 1988; Falcke & Biermann 1996) and we will employ it here as well. The underlying assumption of the model is that the jet is described well by a freely expanding flow with roughly uniform velocity, implying a constant half-opening angle ϕ. Such a description should be a good first order approximation at least in a limited region around the location where most of the radio emission originates. In such a jet, the magnetic field evolves quickly into an almost purely toroidal topology and follows $B \propto z^{-1}$, where z is the distance along the jet, measured from the jet origin at the compact object.

Rather than assuming adiabatic evolution of the particle distribution, the BK model imposes fractional equipartition between particles and magnetic fields such that the particle pressure is $p_{\rm part} = \frac{p_{\rm mag}}{\xi_{\rm B}} \propto z^{-2}$ is a fixed fraction of the magnetic pressure $p_{\rm mag}$, where $\xi_{\rm B}$ is the equipartition fraction between particles and the magnetic field[1]. One may allow for the presence of protons with a contribution $p_{\rm prot} = \xi_{\rm p} p_{\rm part}/(1+\xi_{\rm p})$ to the particle pressure. The electrons (and possibly positrons, subsumed in the non-proton part of the particle pressure) are assumed to follow a powerlaw distribution with index s such that the number density follows $n(\gamma) = C_{\rm e}\gamma^{-s}$. For simplicity and lack of better measurements in the case of Cygnus X-1 we will use $s = 2$, which is the fiducial value often used in AGN jets. We will also assume that, due to Doppler boosting, the emission is entirely dominated by the approaching jet, which is supported by the lack of emission detected from the counterjet in Cygnus X-1. Finally, we assume that the synchrotron emitting and absorbing plasma has a volume filling fraction of $f \leq 1$.

The synchrotron emissivity, measured in the observer's frame, is given by

$$j_\nu = \frac{2.4\times10^{-17}\,{\rm ergs}}{{\rm cm^3\,Hz\,s}}\frac{2p^{7/4}\xi_{\rm B}^{3/4}\delta^2 f}{1+\xi_{\rm p}}\left[\frac{2}{1+\xi_{\rm B}}\right]^{\frac{7}{4}}\left[\frac{8.4\,{\rm GHz}}{\nu}\right]^{\frac{1}{2}} \equiv C_0\,p^{\frac{7}{4}}\delta^2 \quad (1)$$

and the self-absorption coefficient, again measured in the observer's frame, is

$$\alpha_\nu = \frac{2.3\times10^{-12}}{\rm cm}\frac{2p^2\xi_{\rm B}}{1+\xi_{\rm p}}\left[\frac{2}{1+\xi_{\rm B}}\right]^2\left[\frac{8.4\,{\rm GHz}}{\nu}\right]^3\delta^2 f \equiv C_1 p^2\delta^2 \quad (2)$$

with the obvious definitions of C_0 and C_1 (Rybicki & Lightman 1979). p is the total (magnetic plus particle) pressure, measured in the jet frame. $\delta \equiv [\Gamma(1-\beta\cos(\theta))]$ is the Doppler factor . The jet velocity $v \equiv \beta c \equiv c\sqrt{1-1/\Gamma^2}$ likely falls into the range of $0.5 < \beta < 0.7$ and the viewing angle falls into the range $25^\circ < \theta < 50^\circ$ (Stirling et al. 2001; Brocksopp et al. 2002; Gleissner et al. 2004). We will use $\beta \sim 0.6$ and $\theta \sim 35^\circ$ as fiducial values, which gives $\delta \equiv 2.5\delta_{1.6}$.

For radio frequencies, the innermost region of the jet is optically thick, not contributing significantly to the total luminosity (see Fig. 1), which allows us to extend the lower integration limit for the luminosity along the jet axis (denoted by the z-coordinate) to $z \to 0$. The luminosity emitted in a frame comoving with the jet plasma is then

$$L_\nu = \int_0^\infty dz\sin(\theta)\int_{-\phi z}^{\phi z} dx\frac{j_\nu}{\alpha_\nu}\left(1-\exp\left(-\tau_\nu\right)\right) \quad (3)$$

for the optical depth $\tau_\nu(z,x) = \frac{2\alpha_\nu}{\sin(\theta)}\sqrt{(\phi z)^2 - x^2}$. We then define the photospheric radius z_0 as the location where the optical depth through the spine of the jet is one, i.e., $\tau_\nu(z_0, x=0) = 1$ and express all distances in units of z_0 and all other quantities relative to their value at z_0. The pressure can then be written as $p = p_0\left(z/z_0\right)^{-2}$ with

$$p_0 = \sqrt{\sin(\theta)/\left(2C_1\delta^2\phi z_0\right)} \quad (4)$$

The jet luminosity from eq. (3) is then

$$L_\nu = 5.1\,z_0^{\frac{17}{8}}[\sin(\theta)]^{\frac{7}{8}}\phi^{\frac{9}{8}}C_0 C_1^{-\frac{7}{8}}\delta^{\frac{1}{4}} \quad (5)$$

As was shown by BK, the emitted spectrum is flat, $\alpha = 0$.

The VLBA flux of the Cygnus X-1 jet is $F_\nu \sim 12\,{\rm mJy}$ (Stirling et al. 2001). For a distance of $D \equiv 2\,{\rm kpc}\,D_2$, the *observed* luminosity is

$$L_{\rm Cygnus X-1} = 5.7\times10^{19}\,{\rm ergs\,s^{-1}\,Hz^{-1}}\,{D_2}^2 \quad (6)$$

Epochs A and C are very similar in their characteristics: About 50% of the flux is resolved along the jet and both have essentially equal total fluxes of 12 and 12.9 mJy and equal angular resolution along the jet. Epoch B has a lower resolved-to-unresolved flux ratio

In epochs A and C of Stirling et al. (2001), 50% of the jet flux is resolved along the jet axis with an angular resolution of $\mu_{50\%} \sim 3$mas, we can go back to eq. (3) and solve for the value of $z_{50\%}$ for which 50% of the jet emission comes from regions with $z > z_{50\%}$. Numerical evaluation (see Fig. 1) shows that this is the case for $z_{50\%} \sim 3z_0$. Given $\mu_{50\%} \sim 3$mas, we can thus determine the photospheric distance z_0:

$$z_0 \sim 5.2\times10^{13}\,{\rm cm}\left(\frac{\sin 35^\circ}{\sin\theta}\right)\left(\frac{\mu_{50\%}}{3{\rm mas}}\right)D_2 \quad (7)$$

Epoch B (which has slightly lower resolution along the jet) shows a more compact jet, with only about 30% of the flux resolved, which would imply a somewhat lower value of z_0 (by about a factor of 2). This might be due to changes in viewing angle due to jet precession, or possible interaction with the powerful wind from the companion (see §3). We will thus carry the dependence on $\mu_{50\%}$ through for transparency. Strictly speaking, eq. (7) is a lower limit on z_0, since the VLBA observations might have resolved out and thus missed some of the larger

[1] $\xi_{\rm B}$ is a free, unconstrained parameter. The rough proportionality between $p_{\rm part}$ and $p_{\rm mag}$ near the radio emission region can be relaxed slightly. However, a strongly non-linear relation between the two would lead to steep synchrotron spectra and is thus ruled out observationally.

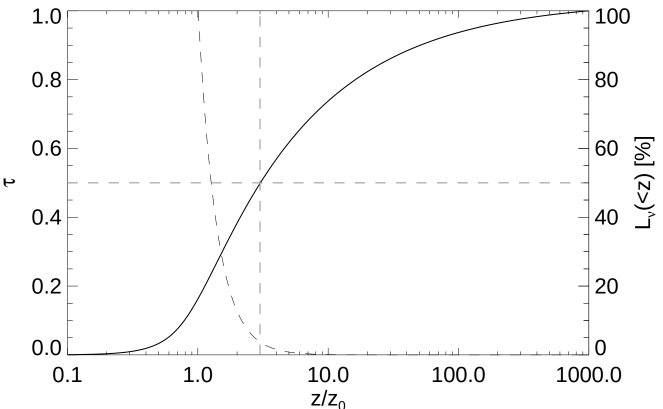

FIG. 1.— Cumulative luminosity profile $L_\nu(< z)$ from the core to z/z_0 (solid curve) and the optical depth to synchrotron-self-absorption across the jet at location $z/z0$ (dashed curve). Also shown is the location of the 50% radius, $z_{50\%}$ (dashed grey lines) at about $z_{50\%} \sim 3z_0$.

scale, low surface brightness emission. However, since the total flux in the VLBA image is close to the total flux seen by other instruments, it is unlikely that z_0 is off by more than a few tens of percent.

It is noteworthy that, from eq. 5, the flux density is

$$F_\nu = L_\nu/(4\pi D^2) \propto (\mu_{50\%})^{\frac{17}{8}}\, D^{\frac{1}{8}}\, (\sin\theta)^{-\frac{10}{8}}\, \delta^{\frac{1}{4}} \qquad (8)$$

i.e., for a given angular size $\mu_{50\%}$, the flux from a compact jet is *almost* independent of the distance D to the source! Because the angular distance and the luminosity distance scale differently with redshift, it should be possible in principle to use jet cores as yard sticks. The main obstacle for such a use is the fact that relativistic beaming will introduce significant uncertainty and bias, such that a statistical analysis of this effect will almost certainly be flawed. However, given a distance estimate, a measurement of F_ν and $\mu_{50\%}$ will yield a constraint on $(\sin\theta)^{-\frac{10}{8}}\,\delta^{\frac{3}{8}}$. Together with independent constraints on θ (e.g., from the jet-counterjet surface brightness ratio), this provides a constraint on the jet velocity, with very little error associated with distance uncertainties, which are dominant in velocity measurements based on superluminal proper motion (Fender 2003).

By equating eqs. (5) and (6) and substituting from eqs. (4) and (7) we can calculate the jet opening angle:

$$\phi \sim 0.06^\circ \left[\frac{\sin\theta}{\sin 35^\circ}\right]^{\frac{10}{9}} \left[\frac{1+\xi_{\rm p}}{\delta_{1.6}^2 2fD_2}\right]^{\frac{1}{9}} \times \left[\frac{3{\rm mas}}{\mu_{50\%}}\right]^{\frac{17}{9}} \left[\frac{F_{8.4{\rm GHz}}}{12{\rm mJy}}\right]^{\frac{8}{9}} \qquad (9)$$

The value we find for ϕ is very small compared to jet opening angles observed in AGN jets (e.g., in the M87 jet, the half-opening angle of the large scale jet is of order $\phi \sim 1.5^\circ$), but entirely consistent with the upper limit of $\phi < 2^\circ$ set by the VLBA observations Stirling et al. (2001). If the jet is in free expansion, this small opening angle would imply a very low internal sound speed, i.e., a pool of cold particles in addition to the synchrotron emitting electrons (see §3). Note that the value of ϕ is rather sensitive to $\mu_{50\%}$, which introduces significant uncertainty given the variance of this value between epochs A through C of Stirling et al. (2001), up to a factor of 5 (the largest value of $\phi \sim 0.3^\circ$ is found for epoch B).

Inserting this into eq. (4) we get

$$p_0 \sim \frac{1\,{\rm erg}}{{\rm cm}^3} \frac{1+\xi_{\rm B}}{2\delta_{1.6}^{\frac{8}{9}}\xi_{\rm B}^{\frac{5}{9}}} \left[\frac{\sin\theta}{\sin 35^\circ}\frac{1+\xi_{\rm p}}{2fD_2}\frac{\mu_{50\%}}{3\,{\rm mas}}\frac{12\,{\rm mJy}}{F_{8.4{\rm GHz}}}\right]^{\frac{4}{9}} \qquad (10)$$

Given the opening angle and pressure of the jet, we can now estimate the kinetic jet power for the leptonic and electromagnetic jet content (i.e., not counting the rest mass kinetic energy in possible thermal particles the jet might be carrying, but including both jet and counterjet):

$$\begin{aligned} W_{\rm min} &\sim 2\times 4p\Gamma^2\beta c\pi(\phi\, z)^2 \\ &\sim 1.9\times 10^{33}\,{\rm ergs\,s^{-1}} \left(\frac{2}{1+\xi_{\rm B}}\right)^{\frac{5}{9}} \Gamma_{1.25}^2\beta_{0.6} \\ &\left[\frac{\sin(\theta)}{\sin(35^\circ)}\left(\frac{12\,{\rm mJy}}{F_{8.4{\rm GHz}}}\frac{3\,{\rm mas}}{\mu_{50\%}}\right)^2 \frac{\xi_{\rm B}\,(1+\xi_{\rm P})}{2fD_2^2\delta_{1.6}^2}\right]^{\frac{2}{3}} \end{aligned} \qquad (11)$$

This estimate is significantly smaller than the $W \sim 10^{35}\,{\rm ergs\,s^{-1}}$ estimated by Spencer et al. (2001)[2] and typical values assumed in the literature for compact XRB jets (Fender et al. 2003). This is mainly due to the smaller opening angle required by the observed value of $\mu_{50\%}$. Unlike the value for ϕ from eq. (9, the W estimate in eq. (11) is less sensitive to the value of $\mu_{50\%}$. The $\mu_{50\%}^{4/3}$ dependence still introduces an uncertainty of up to a factor of 3, given the variance in $\mu_{50\%}$ and $F_{8.4GHz}$ seen between the three different epochs of Stirling et al. (2001), with epoch B leading to a power estimate of $W_{\rm min} \sim 6\times 10^{33}\,{\rm ergs\,s^{-1}}$.

3. DISCUSSION

At face value, this result implies that the kinetic jet power carried by fields and radio emitting electrons is smaller than the radiative power in X-rays by about 4 orders of magnitude. If other X-ray binary jets in the low–hard state were similar in nature to the jet in Cygnus X-1 , this would imply that jets do *not* carry away most of the accretion power in low–hard state and quiescent sources, as recently suggested by Fender et al. (2003). This would require an inefficient accretion scenario like an ADAF (Narayan & Yi 1994), CDAF (Quataert & Gruzinov 2000), or ADIOS (Blandford & Begelman 1999). It would also imply that recent, independent estimates of the jet power from steady low/hard state jets (Fender et al. 2005; Heinz & Grimm 2005) are off by four orders of magnitude.

Such a low power outflow in Cygnus X-1 is in *direct contradiction* to the energy requirement for the recently found thermal shell around the putative radio lobe of Cygnus X-1 (Gallo et al. 2005), which requires an average jet power of

$$3\times 10^{36}\,{\rm ergs\,s^{-1}} < W_{\rm shell} < 3\times 10^{37}\,{\rm ergs\,s^{-1}} \qquad (12)$$

Furthermore, several low/hard state XRBs show IR excesses that have been convincingly interpreted as

[2] semi-analytic models by Markoff et al. (2001) also give significantly larger values for the power.

jet emission (XTE J1118+480, Fender et al. 2001, GX 339-4, Homan et al. 2005; Nowak et al. 2005, GRS 1915+105, Ogley et al. 2000) with total radiative energy output from the jet in excess of 5% of the total radiative output, amounting to $L_{\rm jet} > 2 \times 10^{35}\,{\rm ergs\,s^{-1}}$ in the case XTE J1118+480 (Fender et al. 2001). The jet power must exceed the radiative power by a significant margin since otherwise the jet would radiate away more energy than it is carrying and thus violate energy conservation[3]. While the bright companion of Cygnus X-1 makes direct observations of an IR/optical contribution by the jet difficult, Fender & Kuulkers (2001) argues that a flat jet spectrum extending to the V-band would still require a kinetic power in excess of $\sim 10^{35}\,{\rm ergs\,s^{-1}}$.

It is thus rather unlikely that the power of the Cygnus X-1 jet is truly as low as implied by eq. (11) for the fiducial choice of parameters. We will discuss four alternative (not necessarily mutually exclusive) conclusions that can be reached from the mismatch between eqs. (11) and (12):

3.1. *Jet composition*

It is clear from eq. (11) that $W_{\rm min}$ depends on $\xi_{\rm B}$ and $\xi_{\rm p}$, thus, jet composition and field strength both factor into the total power.

It is straight forward to show that the a significant deviation from equipartition between particles and magnetic field will *not* change the estimate of eq. (11) anywhere close to what is needed to bring it in line with the limits from Gallo et al. (2005). The asymptotic dependence on $\xi_{\rm B}$ is as follows: for small $\xi_{\rm B}$, $W_{\rm min}$ actually *decreases* rather than increasing, thus exacerbating the power discrepancy. For large $\xi_{\rm B}$ on the other hand, $W_{\rm min} \propto \xi_{\rm B}^{1/9}$, requiring $\xi_{\rm B} \sim 10^{36}$ to bring $W_{\rm min}$ in line with the results by Gallo et al. (2005). Such large values are clearly unreasonable and unphysical. In other words: we cannot constrain the magnetic field strength, but significant deviations from equipartition will also *not* bring eq. (11) in line with eq. (12).

On the other hand, if the jet contains a large amount of cold, thermal plasma, the kinetic power can be significantly enhanced. Before going into more detail, it is important to note that simple charge balance of cold protons with the observed synchrotron emitting electrons will not enhance the power by more than two orders of magnitude because the above estimate is based on minimum energy arguments. For $\xi_{\rm B} \sim 1$, the Lorentz factor of the electrons contributing most of the 8.4 GHz flux is about $\gamma_{\rm e,8.4GHz} \approx 70\left(p_0/1\,{\rm erg\,cm^{-3}}\right)$, thus adding one cold proton per emitting electron would only raise the kinetic power by about an order of magnitude, insufficient for bringing it in line with eq. (12).

The kinetic jet power including the inertial term is

$$W = W_{\rm min} + 2\rho c^3 \Gamma\beta\left(\Gamma - 1\right)\pi\left(\phi z\right)^2 \tag{13}$$

where $w = \gamma/(\gamma-1)p + \rho c^2$ is the enthalpy. The amount of cold gas necessary to bring the power estimate in line with the large scale constraints from eq. (12, parameterized as $W_{\rm tot} = 3 \times 10^{36}\,{\rm ergs/s} W_{36.5}$, by Gallo et al. (2005) is then

$$\rho_0 \gtrsim \frac{2.4 \times 10^{-17}\,{\rm g}}{{\rm cm^3}}\left[\frac{0.06^\circ}{\phi}\right]^2 W_{36.5}\frac{3/16}{\Gamma\beta(\Gamma-1)} \tag{14}$$

or about 2000 protons per radio emitting electron (we have abbreviated the dependence on the underlying parameters $\xi_{\rm B}$, $\xi_{\rm p}$, and f in the opening angle ϕ - see eq. (9) for the detailed parametric dependence).

This is a firm lower limit. We should note again that the question whether jets contain large amounts of protons has been discussed in the literature many times over but remains essentially unanswered to this day, especially in the newer and less well studied class of XRB jets. Any new constraint on an individual object, like the one derived here, will thus be valuable addition to the debate. Note also that energetic requirements for the presence of cold, thermal protons (derived here) are different from the requirement of a thermal parent population of leptons at the base of the jet, as required in models that attempt to match not only the radio but also IR through X-ray observations of X-ray binaries, such as those by (Markoff et al. 2001; Markoff & Nowak 2004; Yuan & Cui 2005). These particles are required on spectral grounds, not for energetic reasons and no quantitative requirement for the presence of cold protons can be made from them.

Arguments for the presence of cold protons *have* been made on dynamical grounds in order to explain the opening angles of jets in the case of freely expanding, ballistic jet models (Falcke & Biermann 1996; Markoff et al. 2001). However, the opening angle and the jet power in those models are free parameters and thus constraints on the presence of protons in the jets are qualitative *only.* We will employ a similar argument in the following paragraphs, based on the new constraint we derived for the jet opening angle in eq. (9).

Above and beyond the lower limit from eq. (14), the total amount of cold plasma that might be traveling down the jet cannot be constrained in the absence of direct radiative signatures, and thus the power enhancement that could be provided by cold particles is formally unconstrained. However, the small opening angle implied by eq. (9) could be interpreted as evidence for the presence of large amounts of cold matter if we assume that the jet opening angle is approximately equal to the Mach cone of the jet[4], i.e.,

$$\phi \sim c_{\rm s}/(\beta c\Gamma) \sim \sqrt{5p/3\rho}/(\beta c\Gamma) \tag{15}$$

Thus, if cold, thermal particles, such as protons (with the appropriate number density of cold electrons for charge balance), are present in the jet and are the reason for the small opening angle, we can use the estimate of ϕ and eq. (15) to obtain an estimate of the rest mass density ρ inside the jet. This is, in effect, an upper limit because

[3] Even the X-ray emission in the low/hard sate systems might partially originate in the jet (Markoff et al. 2001; Markoff & Nowak 2004), in which case the power requirement would increased by another 2 orders of magnitude, though synchrotron emission is probably not the dominant source of the X-rays (Heinz 2004).

[4] Note that in a BK type jet the "temperature" (p/ρ) of the relativistic plasma is constant along the jet, thus defining a temperature based on the jet opening angle is well posed and meaningful. However, even if the thermal plasma were not of constant temperature along the jet because of radiative cooling, this argument would still hold, as the opening angle would define the temperature at the location where the jet becomes ballistic (i.e., inertia dominated). Further out, the thermal temperature might be lower than at that point, but the opening angle would remain at its ballistic value.

the jet might be partially collimated by magnetic fields and thus not in free, ballistic expansion.

$$\rho_0 \lesssim \frac{3.2\times10^{-15}\,\mathrm{g}}{\mathrm{cm}^3}\frac{p_0}{1\,\mathrm{ergs\,cm^{-3}}}\left(\frac{0.06^\circ}{\phi}\frac{0.75}{\Gamma\beta}\right)^2 \quad (16)$$

The jet power is then

$$W \lesssim 3\times10^{38}\,\mathrm{ergs\,s^{-1}}\frac{\rho_0}{4.5\times10^{-16}\mathrm{gcm^{-3}}}\frac{\Gamma\beta(\Gamma-1)}{3/16} \quad (17)$$

In this case, the high particle densities of $n \sim 2\times10^9\,\mathrm{cm^{-3}}$ might imply a detectable thermal X-ray flux from these particles (as is the case in the jet of SS433). The temperature of this thermal gas would be $T = 2.3\times10^6\,\mathrm{K}(\phi/0.06^\circ)^2(\beta\Gamma/0.75)$. The total thermal 0.5-10 keV luminosity from the jet would be $L_{\rm brems} \approx 8\times10^{37}\,\mathrm{ergs\,s^{-1}}$, comparable to the bolometric luminosity believed to be coming from the accretion flow. The associated soft X-ray/UV/optical line emission, which would be very similar to the spectrum of SS433, has not been observed.

Such a dense plasma would cool rapidly[5] if it were present from the very base of the jet (the cooling time there would be about 10^{-9} seconds, much shorter than the dynamical time). In order to avoid catastrophic radiative cooling, particles would have to injected at a large distance from the black hole, $z_{\rm cool} \gtrsim 10^{13}\,\mathrm{cm}$ (where the radiative cooling time is longer than the dynamical time). Coincidentally, this is comparable to the orbital separation of $\sim 10^{12}\,\mathrm{cm}$ of the system.

It would be natural, then, to interpret the presence of such a putative thermal component as mass loading from the powerful wind blown by the companion. Since this thermal plasma is supposed to set the opening angle of the jet, collimation and acceleration would still have to be happening at the location where the mass loading takes place, which is very far from the black hole. This would provide interesting asymptotic constraints for models of jet dynamics.

3.2. *Low filling factor*

Another way to increase the power estimate from eq. (11) would be a filling factor f much smaller than one. While the opening angle ϕ depends only weakly on f, the jet power is somewhat more sensitive. A very low filling factor would translate to a physical picture where small regions dissipate a large amount of energy into relativistic particles, such as would occur in magnetic reconnection events in a turbulent flow.

In this context, it is interesting to note that high resolution radio images of AGN jets (which have are much better suited for high resolution imaging due to their larger ratio of object size to distance) show very non-uniform and sometimes unresolved, filamentary emission at least in the regions where the jet is optically thin (e.g. Biretta et al. 1995). It might well be that the emission from the smaller, optically thin region is similarly filamentary.

In order to bring the estimate of eq. (11) back in line with the macroscopic limits (Gallo et al. 2005), the filling factor would have to be $f \lesssim 3\times10^{-5}$, which is very small indeed and difficult to achieve with internal shock scenarios (Yuan & Cui 2005) or multi-zone continuous emission models (Markoff et al. 2001).

3.3. *A non-radiative source for the kinetic power*

It is also possible that the flat spectrum, steady, low/hard state jet that is observed directly in the VLBA radio images is *not* the source of the power that drives the ISM shell.

For example, it is possible that the radio emission originates in a spine along the jet axis, which is embedded in a much wider sheath of low emissivity plasma with opening angle $\phi_{\rm sheeth}$. In this case, the true kinetic power could be much larger (by a factor of $(\phi_{\rm sheeth}/\phi)^2$. To bring the power estimate in line with the value from Gallo et al. (2005), we would require $\phi_{\rm sheeth} \geq 2^\circ$. Note that this is different from a low filling factor scenario since the optical depth through the spine is larger than that through a low filling factor jet of equal emission measure. Why most of the flow would be non-radiative in this case is similarly difficult to understand as the case of a low filling factor, however. It would amount to postulating an unrelated, much larger angle flow that would supply all the power but no radiation and, in essence, would mean that the actually observed jet has no physical relation to the large scale bubble observed around Cygnus X-1. Whether a radiative signature of such a larger angle outflow exists and how it would tie in with ADIOS and disk wind models is not clear.

Since such a putative outflow has so far not been observed, there are no constraints on the nature of the flow other than the kinetic power one would have to postulate based on the large scale requirements. It would thus be impossible to derive mass flow rates and outflow velocities. However, given the observed power and using the Eddington accretion rate of $\dot{M} = 1.5\times10^{19}\,\mathrm{g/s}(M/10M_\odot)$ for a 10 solar mass black hole as an upper limit on the mass flow rate, we can conservatively put a lower limit of $v_{\rm sheeth} > 6000\,\mathrm{km\,s^{-1}}W_{36.5}^{1/2}$ on the outflow velocity.

It is also possible that most of the kinetic power in the Cygnus X-1 jets is supplied by the type of optically thin radio outbursts observed in many Galactic XRB sources like, for example, GRS 1915+105 (Mirabel & Rodríguez 1994; Fender et al. 1999) rather than the steady compact jets discussed here. This would have interesting consequences for the estimates of duty cycles and radiative efficiencies for such events. Since they are unrelated to the VLBA observations discussed here, we will not discuss this possibility in more detail.

It should be noted that the flat spectrum IR emission from low/hard state XRBs that has been interpreted successfully as jet emission still places power constraints on the jet that, while smaller than eq. (12), still far exceed that of eq. (11). IR observations of Cyg X-1 are difficult because of the high Galactic background and the bright

[5] It should be noted that a fixed opening angle in a cooling jet is *not* a contradiction: the opening angle of a ballistic jet is set at the point where the jet goes out of transverse causal contact, i.e., where the sideways expansion becomes faster than the effective internal sound speed of the jet plasma (e.g., the fast magnetosonic speed in a magnetized jet). Beyond that point, the jet will expand at an essentially fixed rate, possibly cooling adiabatically and/or radiatively. In other words, the temperature does not need to be proportional to ϕ^2 everywhere along the jet. Thus, contrary to a first glance educated guess, no mechanism is necessary to reheat the plasma to maintain a fixed opening angle.

companion, thus we cannot apply these limits directly. However, if the Cyg X-1 jet exhibits IR fluxes similar to those from other low/hard state jet sources, an unrelated source of kinetic power that might be responsible for the large scale ISM shell could not explain the tight IR/radio correlation, and thus the constraints derived in §3.1 and 3.2 would hold. Ongoing Spitzer IR monitoring will help to answer this question.

3.4. *Grossly different kinematics*

Finally, the treatment used in this paper might not be applicable because some of the basic assumptions we employed might be false. This would imply that the most commonly used description for jet cores that captures the critical observable features, the "Blandford/Koenigl" model, would grossly misrepresent the kinematics and emissivity characteristics of jet cores (by many orders of magnitude). This in itself would be an important constraint on jet modeling. We shall briefly consider whether a :

The most obvious deviations from a BK model would be (a) a non-conical jet, (b) a deviation from the assumed $p \propto z^{-2}$ scaling, or (c) an electron spectral index s different from $s = 2$. Barring seriously pathological jet geometries and kinematics, the jet should be well described (at least near the radio photosphere) by powerlaws for the jet radius $R = R_0(z/z_0)^\zeta$ (with $0 \leq \zeta \leq 1$ for collimation) and the pressure $p = p_0(z/z_0)^\chi$. Following eq. (17), the kinetic power of the jet is $W = \left[4p\Gamma^2\beta c + \rho\left(\Gamma^2 - \Gamma\right)\beta c^3\right]\pi R^2$. If the jet is *not* cold matter dominated ($\rho c^2 \ll p$), the energy flux is only constant along the jet if $\chi = -2\zeta$, which we impose as a condition in the following. If it *is* cold matter dominated, once again §3.1 applies.

The spectral index for such a jet would be $\alpha = (18\zeta - 8 - 2s - 3s\zeta)/12\zeta$. Since the observed spectrum in low/hard state XRB jets is flat to slightly inverted (in the case of GX 339-4, $\alpha \sim 0.1$, while at low frequencies, the jet in XTE J1118+480 is rather steep, $\alpha \lesssim 0.5$), we can impose $\alpha \geq 0$ and find $s \leq (18\zeta - 8)/(2 + 3\zeta) \leq 2$. Fermi acceleration, which is often invoked as the origin of the electron powerlaw spectra, typically produces spectra with $s \geq 2$, which is consistent with the above condition *only* for the case $s = 2$ and $\zeta = 1$, *i.e.*, a BK jet. Slightly smaller values of s *might* be possible to achieve with relativistic Fermi acceleration (Kirk & Heavens 1989) or reconnection, but the parameter space is severely limited to a region around the classical BK model ($s = 2$ and $\zeta = 1$). Estimates presented in this paper will not be affected too strongly by such a small deviation in parameters.

One might also argue that the assumption of uniform velocity is incorrect, i.e., Γ might be increasing significantly inside the radio emission region through active hydrodynamic or MHD driving, in which case a simple kinematic model would be inappropriate. It is beyond the scope of this paper to present dynamical jet models in any detail. However, it is clear that the radio emission region is very far away from the black hole, roughly 10^7 gravitational radii according to eq. (7). Jet acceleration at such distances would be very surprising indeed, presenting serious asymptotic challenges to dynamical jet models. Significant acceleration over such large scales would also imply enormous terminal Lorentz factors which would be at odds with the observational velocity constraints on Cygnus X-1 (Gleissner et al. 2004).

Thus, it is not trivial to construct a jet model that produces a flat spectrum, obeys energy conservation, has reasonable terminal velocities, and still deviates significantly from a BK-jet.

However, it is worth keeping in mind that, while the Cygnus X-1 jet is generally well described by a BK jet, it does exhibit some features that go beyond this simple, robust picture: The jet does show signs of curvature in epoch A of Stirling et al. (2001), which might be due to precession induced by the binary orbit, possibly explaining the different appearance of epoch B compared to epochs A and C. Furthermore, other Galactic jet sources show an even broader variety of behaviors, such as spectral pivoting (Corbel et al. 2000) and short term variability. Such behavior clearly requires more complex models, including the possibly confining and contaminating presence of stellar winds (which, interestingly, should affect the Cygnus X-1 jet most strongly, given the powerful wind from it O9.7 Iab companion). Finally, the question of what accelerates the relativistic electrons inside the jet (as postulated by all jet emission models), which directly relates to the assumed quasi-equipartition between particles and magnetic field in the BK model, remains unanswered.

It is beyond the scope of this paper to present models addressing these issues, but it is possible that more complex dynamical models, incorporating sophisticated particle transport calculations, will lead to a significantly different power estimate that is more in line with the observed values from eq. (12). As it stands, the requirement to satisfy the observed flat spectrum and the VLBI surface brightness distribution (in detail), while at the same time satisfying the large scale power requirements, presents a benchmark for new jet models to measure up to.

4. SUMMARY

We presented a detailed analytic model of the VLBA emission from the Cygnus X-1 radio jet to derive a parametric expression of the jet power $W_{\rm jet}$. By applying this expression to new, independent observational constraints on $W_{\rm jet}$ by Gallo et al. (2005), we derived stringent constraints on fundamental parameters of the jet, which lead us to the following set of alternative conclusions: The jet must either (a) contain large amounts of cold protons and/or (b) have am extremely low filling factor, and/or (c) the power that drives the ISM shell is not carried by the compact VLBA radio jet but some other, unrelated type of outflow (e.g., optically thin radio flares or a broad accretion disk wind) and/or (d) a strong asymptotic deviation from the analytic Blandford-Koenigl model we employed (however, still subject to the very restrictive spectral and imaging constraints provided by the data). Each of these alternatives presents valuable input for jet modeling efforts.

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